Topic: Even-odd identities

Question: Find the values.

$\sin(-855°)$ and $\cos(-855°)$

Answer choices:

A $\sin(-855°) = \frac{\sqrt{3}}{2}$ and $\cos(-855°) = -\frac{1}{2}$

B $\sin(-855°) = -\frac{1}{\sqrt{2}}$ and $\cos(-855°) = \frac{1}{\sqrt{2}}$

C $\sin(-855°) = \frac{1}{2}$ and $\cos(-855°) = -\frac{\sqrt{3}}{2}$

D $\sin(-855°) = -\frac{1}{\sqrt{2}}$ and $\cos(-855°) = -\frac{1}{\sqrt{2}}$

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Solution: D

Note that

$$855^\circ = 720^\circ + 135^\circ = 2(360^\circ) + 135^\circ$$

Therefore, an angle of 855° differs from an angle of 135° by an integer multiple of 360° (i.e., by an integer number of "full turns" about the origin). Therefore,

$$\sin(855^\circ) = \sin(135^\circ) \text{ and } \cos(855^\circ) = \cos(135^\circ)$$

Also,

$$135^\circ - 90^\circ = 45^\circ = 180^\circ - 135^\circ$$

Thus an angle of 135° is in the second quadrant, and it lies "halfway between" an angle of 90° and an angle of 180°. Since the sine of every angle in the second quadrant is positive and the cosine of every angle in the second quadrant is negative, we have the following:

$$\sin(135^\circ) = \frac{1}{\sqrt{2}} \text{ and } \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

Using even-odd identities:

$$\sin(-855^\circ) = -\sin(855^\circ) = -\sin(135^\circ) = -\frac{1}{\sqrt{2}}$$

and

$$\cos(-855^\circ) = \cos(855^\circ) = \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

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Topic: Even-odd identities

Question: Find the values.

$$\tan\left(-\frac{5\pi}{6}\right) \text{ and } \sec\left(-\frac{5\pi}{6}\right)$$

Answer choices:

A $\tan\left(-\frac{5\pi}{6}\right) = \sqrt{3}$ and $\sec\left(-\frac{5\pi}{6}\right) = \frac{2}{\sqrt{3}}$

B $\tan\left(-\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$ and $\sec\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

C $\tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$ and $\sec\left(-\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

D $\tan\left(-\frac{5\pi}{6}\right) = -\sqrt{3}$ and $\sec\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2}$

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Solution: C

Note that an angle of $5\pi/6$ radians is in the second quadrant, an angle of $\pi/6$ radians is in the first quadrant, and the reference angle for both $5\pi/6$ and $\pi/6$ is $\pi/6$. Recall that both the sine and the cosine of every angle in the first quadrant are positive, that the cosine of every angle in the second quadrant is negative, and that the sine of every angle in the second quadrant is positive. Thus by the reference angle theorem,

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

and

$$\sin\left(\frac{5\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

Since the cosine function is even,

$$\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

And since the sine function is odd,

$$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$$

Also, the tangent function is odd, so

$$\tan\left(-\frac{5\pi}{6}\right) = -\tan\left(\frac{5\pi}{6}\right) = -\frac{\left(\sin\frac{5\pi}{6}\right)}{\left(\cos\frac{5\pi}{6}\right)} = -\frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

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The secant function is even, so

$$\sec\left(-\frac{5\pi}{6}\right) = \sec\left(\frac{5\pi}{6}\right) = \frac{1}{\cos\left(\frac{5\pi}{6}\right)} = -\frac{2}{\sqrt{3}}$$

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Topic: Even-odd identities

Question: Find the values.

$$\csc\left(-\frac{49\pi}{3}\right) \text{ and } \tan\left(-\frac{49\pi}{3}\right)$$

Answer choices:

A $\csc\left(-\frac{49\pi}{3}\right) = -\frac{2}{\sqrt{3}}$ and $\tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

B $\csc\left(-\frac{49\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ and $\tan\left(-\frac{49\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

C $\csc\left(-\frac{49\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

D $\csc\left(-\frac{49\pi}{3}\right) = \frac{2}{\sqrt{3}}$ and $\tan\left(-\frac{49\pi}{3}\right) = \frac{1}{\sqrt{3}}$

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Solution: A

Note that

$$\frac{49\pi}{3} = \frac{(48+1)\pi}{3} = 16\pi + \frac{\pi}{3}$$

Thus an angle of $49\pi/3$ radians differs from an angle of $\pi/3$ radians by an integer multiple of 2π, so we have the following:

$$\sin\left(\frac{49\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

and

$$\cos\left(\frac{49\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Using even-odd identities, we obtain the following:

$$\csc\left(-\frac{49\pi}{3}\right) = -\csc\left(\frac{49\pi}{3}\right) = -\frac{1}{\sin\left(\frac{49\pi}{3}\right)} = -\frac{2}{\sqrt{3}}$$

and

$$\tan\left(-\frac{49\pi}{3}\right) = -\tan\left(\frac{49\pi}{3}\right) = -\frac{\sin\left(\frac{49\pi}{3}\right)}{\cos\left(\frac{49\pi}{3}\right)} = -\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\sqrt{3}$$

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